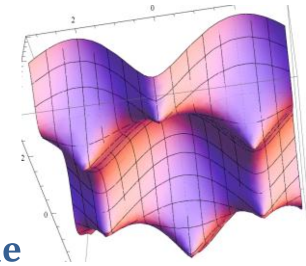

Challenge 3 – Dispersion of Graphene

In our meeting two weeks ago we talked about the graphene. It is interesting how the dispersion relation between E and k is **linear** for low energies at some special momenta. Because of this linear dispersion relation, electrons and holes behave like relativistic particles described by the Dirac Equation.

The above picture may have appeared in your (or Mark's) wildest dreams—when you walking on California Blvd, someone suddenly pull out a gun, pointing at you head, and shouted to you: answer this following question!

1. **As an interesting warm up-question, without looking up any resources, just based on the information above (or in case you remember it anyway…) can you guess <u>the order of magnitude of the velocity, and the energy of an electron in the linear dispersion range</u>?**

Prof. Refael originally changed my question of energy to the speed of an electron in linear dispersion range. But I insist that energy is more interesting to think about. Maybe you want to make an analogy with photon.

2. **Because of the interesting dispersion relation of graphene, there will be a hexagonal shape appearing in the E-kx-ky plot for particular slices of constant E. <u>Please try to make a plot of E-kx-ky for graphene, and find out for what value of kx, ky does this shape appear? What is the value of E at that point?</u>**

$$H_{\mathrm{g-2D}} = \begin{bmatrix} 0 & f(k) \\ -f^{\dagger}(k) & 0 \end{bmatrix} .$$

*Hint: We use tight binding-approximation in this case. The formal way is to solve the Hamiltonian of the two carbon atoms in the graphene unit cell. The Hamiltonian is as above. f(k) is the vector sum of the 3 factors (e^(i **k *Δ r**)(of course, k and Δ r are vectors) with Δ r corresponding to the relative locations of a site in the A sublattice and the 3 B sites neighboring it, and similarly for B and the 3A neighbors for* f^{+}*. Solve for the eigenvalues of the Hamiltonian and you will get the energy values as a function of kx and ky. Notice you need to put in some factor to make the unit right! The factor should be similar to what you did in problem 1.*

3.(extra) If you are interested, you can try instead of 3 wave vectors, a 5 wave vector situation. You might find something we have discussed before!