

FIG. 1: The aftermath of the Chinese satellite interception. The red orbits show tracked debris.

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Caltech Physics League - Round 1 Space Warfare with Leftovers

On January 11th, 2007, China shot down one of its defunct Fengyun weather satellites, using an anti-satellite interceptor moving in the opposite orbit (see, e.g., http://en.wikipedia.org/wiki/2007_Chinese_anti-satellite_missile_test). The success of this experiment was a rather remarkable technological achievement of China.

Apart from the concern over the 'militarization' of space, many raised a more realistic concern: the pollution such experiments leave behind. Currently, there are more than 600,000 objects bigger than 1cm orbiting the Earth, which can be described as 'space junk'. The US strategic command tracking 13000 of these.

The theme of this round of the CPL is the interception and debris physics of the Chinese satellite experiment. The target satellite was a $m_0 = 750\text{kg}$ satellite stationed at a circular orbit altitude of $h_0 = 865\text{km}$. The satellite was shot down using a 'kinetic kill vehicle' orbiting at the same orbit, but in the opposite direction. After scouring the web, I found little information about the kill vehicle; let us assume that it was a chunk of metal of mass $m_k = 50\text{kg}$, and that the destruction of the satellite was solely due to the head-on collision with the kill vehicle.

More information. According to some accounts (see http://en.wikipedia.org/wiki/Space_debris), the collision produces about: 10^6 particles of size $d > 1\text{mm}$, out of which 35000 pieces had $d > 1\text{cm}$, and about 2300 had $d > 5\text{cm}$. Let us approximate the event as producing a total of 10^6 pieces of junk, of sizes varying between 1mm and 0.5m . Let us make the further simplifying assumption that the satellite and kill vehicle were all made of Aluminum alloy, with specific mass $\rho_{AL} \approx 2500\text{kg}/\text{m}^3$. As usual, let us also make the 'spherical cow' approximation, and assume that all debris, and the destroyed satellite are perfectly spherical.

Reading about this experiments, I was wondering - how many of these debris actually stay in orbit? How many fall into the atmosphere? How long do debris stay in space? You will attempt to find answers to exactly these questions.

I. THE COLLISION AND ITS IMMEDIATE AFTERMATH

Let's begin with a simple kinematics warm up.

1. What is the velocity of the satellite and the kill vehicle at the orbits described?
2. Suppose the satellite and kill vehicle were to just recombine to a single body (plastic collision), what would be the apogee and perigee (highest and lowest) points of the resulting hunk of metal?
3. Now let us address the actual event.

The collision actually produced numerous debris. Let us assume that the satellite and kill vehicle were completely destroyed in the collision, and that *in the center of mass frame* the debris resulting from the collision emanate

from the two colliding bodies in a spherically symmetric fashion. To clarify further - as if the combined satellite - kill-vehicle hunk of metal from 2 were to explode and send off debris equally in all directions.

Furthermore, assume that the kinetic energy lost in the collision, is then equally distributed between the debris (a gross overestimate...).

4. For debris of diameter d , what is the emission velocity in the COM frame of the collision? Calculate this number for the smallest and largest pieces resulting from the collision.
5. Clearly big chunks will deviate the least from the original orbit. But small chunks may change their trajectory enough so that they fall directly to Earth. By falling to Earth, I mean they reach altitude of 150km, where the atmosphere begins to thicken up. What is the biggest size chunks that fall to the Earth?
6. Most of the large hazardous pieces surviving in space will be of the diameter range which can not fall to Earth. Let us estimate the 'beam size' of the cloud of such big remains. What is the highest point reached by the particles of the diameter found in 5? What is the maximum lateral width (horizontal spread of debris)? For the latter, find the best estimate.
7. What is the highest point reached by (the smallest) debris?
8. For the smallest particles emitted, *estimate* the fraction falling to Earth.

II. DEBRIS LIFETIME

Above you found the extent of the pollution due to the Fengyun's debris. Let us now find out how long these debris remain in orbit. Even above an altitude of 100km, satellites and debris encounter a very thin atmosphere, which will eventually take them down.

The mass density of the upper atmosphere is strongly fluctuating due to solar activity, but in Fig. 2 you can see an average density profile. As it turns out, to logarithmic accuracy, one can approximate the lower atmosphere, $0 < h < 150km$, as an N_2, O_2 isothermal atmosphere, at a rough temperature of $T_{LA} = 250K$.

For the upper atmosphere, with $150km < h < 1000km$, we can use a similar approximation. The dominant element in the upper atmosphere is atomic Oxygen, whose number density at $h = 150km$ is $n_O = 2.3 \cdot 10^{16}m^{-3}$. The temperature in the upper atmosphere quickly saturates at $T_{UA} = 1000K$. Therefore, assume that the mass density at $h > 150km$ is given by:

$$\rho(R = R_e + h) \approx m_{ON} n_O e^{-(E_p(h) - E_p(150km))/k_B T_{UA}} = m_{ON} n_O e^{GMm_O(\frac{1}{R} - \frac{1}{R_e + 150km})/k_B T_{UA}} \quad (1)$$

with $m_O = 8 \cdot 1.66 \cdot 10^{-27}kg$, the atomic mass of Oxygen, and $k_B = 1.38 \cdot 10^{-23}J/K$ the Boltzmann constant. R is the radius probed, and $R_e = 6400km$ is the Earth's radius. G and M are the gravitational constant and the Earth's mass respectively.

1. What is the typical velocity of an Oxygen atom in the upper atmosphere? How does it compare to the speed of an orbiting satellite?

When an object moves through a gas much faster than the speed of the individual gas molecules, one can approximate the gas particles being at rest. Then the friction a body feels is:

$$F = C_D \rho A v^2 \quad (2)$$

with v the velocity of the body, A its cross section area, ρ the mass density of the gas, and C_D is a geometrical coefficient that depends on the shape of the body.

2. Calculate C_D for a sphere.
3. Assuming that once a satellite enters the lower atmosphere, it quickly slows down and burns, estimate the lifespan of a typical spy satellite, orbiting at $h = 320km$, mass of $m = 2300kg$, and a spherical body of diameter $2.5m$ (for more realistic dimensions see, e.g., http://en.wikipedia.org/wiki/USA_193). Assume that the satellite completes many revolutions around the Earth before reaching $h = 150km$. [If you wish to refine your answer (given the mismatch between our approximate atmospheric mass density and the measured one), note that the temperature in the range $h \sim 300km$ could be taken to be closer to $T_{UA(low-part)} \sim 800K$ (see Fig. 2b)].

Back to the Chinese experiment:

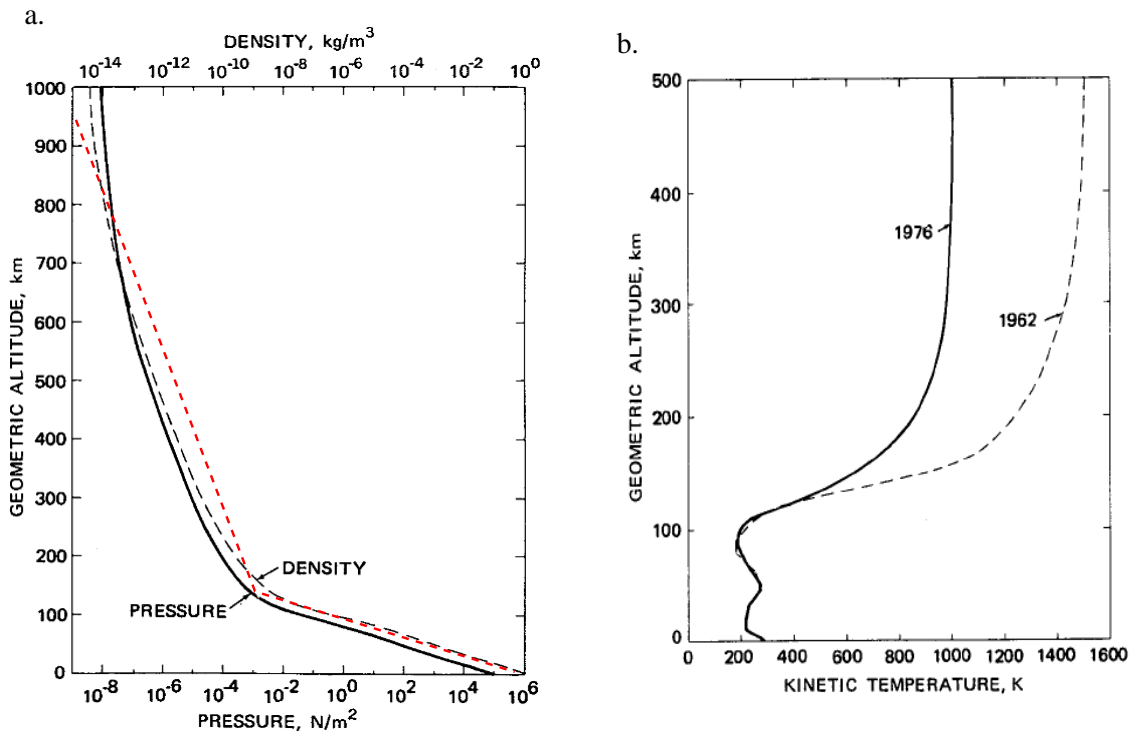


FIG. 2: (a) The density and pressure profiles of the atmosphere. The red dotted line is the approximation we use for the mass density. As you can see, it gets the density up to an order of magnitude. (b) Temperature profile of the upper atmosphere. As you can see, the temperature is highly fluctuating due to solar activity. We will use the 1976. Plots taken from NASA, US Standard Atmosphere, 1976.

4. What would have been the lifetime of the Fengyun weather satellite shot down by china? Assume the satellite is a spherical body made of Aluminum alloy? What is the lifespan of a debris of the satellite with diameter d at the same orbit? Calculate the latter for the smallest debris.

Some debris are particularly worrisome since they orbit in the vicinity of the International Space Station, which is at $h_{ISS} = 350km$. Consider a debris with diameter $d = 2cm$ which is at an elliptic orbit with $R_a = 850km$ and $R_p = 350km$.

- 5. How much energy, and angular momentum is lost in each revolution?
- 6. What is the leading behavior of the eccentricity of the orbit?
- 7. Finally, Estimate how long before the debris hits the lower atmosphere.

Unsolicited advise: note that different parts of the orbit dissipate dramatically differently; don't try to find a precise answer - make reasonable approximations and simplifications to get to within an order of magnitude.

Note that an elliptic orbit is given by:

$$\frac{1}{R(\theta)} = \frac{1}{R_0}(1 + e \cos \theta) \tag{3}$$

with

$$e = \frac{R_a - R_p}{R_a + R_p} \tag{4}$$

the eccentricity of the orbit, and R_0 a constant length-scale, and θ the polar angle of the orbit.

III. USEFUL INFORMATION

Earth's radius:

$$R_e = 6400km. \quad (5)$$

Earth's acceleration of gravity at R_e :

$$g = 9.8m/s^2 \quad (6)$$

Nucleon mass:

$$m_p = 1.66 \cdot 10^{-27}kg \quad (7)$$

Let me know if more info is necessary, in your opinion.