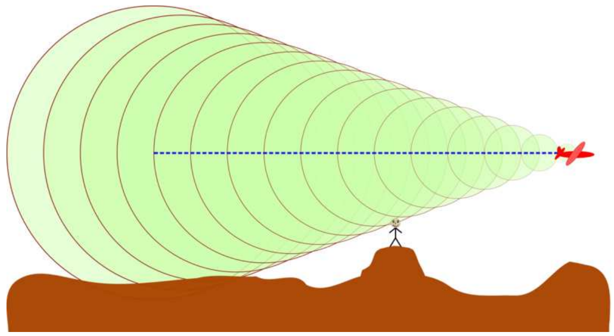

FIG. 1: A schematic diagram of the shock wave emanating from an overhead airplane. The angle θ is the angle between the blue line of flight, and the sides of the expanding cone. (figure taken from $http://mistere9.blogspot.com/2008_04_01_archive.html$)

Caltech Physics League - Round 2

Sonic Booms

The speed of sound was never such a tough barrier for mankind: the crack of a whip is the result of the whip's tip breaking the speed of sound (according to wikipedia, at least). From that to the Concord (or the black-bird spy plane, for that matter), it was mere engineering.

A sonic boom is created when an object moves through a medium faster than the speed of sound in it. A common diagram is given in Fig. 1. A shock wave emanates from the moving object, forming a cone centered on the line of motion. The shock wave has enhanced pressure in it, which in air could be seen, surprisingly due to water condensation, as in Fig. 2. When this shock wave hits a listener, the enhanced pressure is registered as a loud thunder.

- As a warm up, what is the shock wave opening angle, θ (see caption of Fig. 1), of an airplane moving with at speed v, higher than the speed of sound c?

- Now consider an airplane flying at a height h, usually between $10km$ and $20km$, with a velocity $v > c$ (Mach number bigger than 1). Since it is moving faster than sound, it produces a shock wave cone. What is the excess pressure in the shock wave as it hits the ground? Construct a **simple** model which will provide an estimate for the excess pressure as a function of the plane's velocity, height, size, and anything else you deem necessary.

  As can be read in http://www.nasa.gov/centers/dryden/news/FactSheets/FS-016-DFRC.html, for instance, it seems that the typical measure for sonic booms on the ground is in few lb/square-feet. What do you get using your model for something resembling a Concord at Mach 1.5, 15km's above the ground?

  Clearly this is a complex phenomena that depends on many parameters. Do not try to be too detailed - shoot for obtaining an answer which is correct to within about an order of magnitude.

I. USEFUL INFORMATION

The speed of sound in an ideal gas is:

$$c = \sqrt{\frac{\gamma k_B T}{M}} \tag{1}$$

where M is the molecular mass of the gas in kilograms ($1.66 \cdot 10^{-27} kg$ times the number of nucleons in the molecule), T is the temperature, $k_B = 1.38 \cdot 10^{-23} J/K$, and $\gamma = C_p/C_V = 7/5$ for diatomic gases. Feel free to look up any other info about the physical properties of air.

FIG. 2: A photo of the shock wave cone created by the airplane supersonic flight. The cloud is water condensation in the shock wave. (photo taken from $http://penguinflight.com/Sonic/$)