Solution

1. Constants for TE effect

We need \( S \Delta T = \Delta V \)

\( T \) is associated with energy as \( q \) is \( V \). To get energy from \( T \):

\[ k_b T \]

To get energy from \( V \):

\[ eV \]

\[ S \times \frac{k_b}{e} = \frac{1.38 \times 10^{-23}}{1.6 \times 10^{-19}} \approx 100 \text{V/k} \]
From Wikipedia:

<table>
<thead>
<tr>
<th>Material</th>
<th>Seebeck coefficient relative to platinum (μV/K)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Selenium</td>
<td>900</td>
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<tr>
<td>Tellurium</td>
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<tr>
<td>Silicon</td>
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<td>25</td>
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<tr>
<td>Molybdenum</td>
<td>10</td>
</tr>
<tr>
<td>Cadmium, tungsten</td>
<td>7.5</td>
</tr>
<tr>
<td>Gold, silver, copper</td>
<td>6.5</td>
</tr>
<tr>
<td>Rhodium</td>
<td>6.0</td>
</tr>
<tr>
<td>Tantalum</td>
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<td>Lead</td>
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<tr>
<td>Aluminium</td>
<td>3.5</td>
</tr>
<tr>
<td>Carbon</td>
<td>3.0</td>
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</tbody>
</table>

Also:

- $\text{SiGe}$: $S = 300 \mu \text{V}$ at $T \sim 1000 \text{ K}$
- $\text{Bi}_2\text{Te}_3$, $(\text{Bi}_2\text{Se}_3)$: $S \sim 100 \mu \text{V}$
  - $\sigma \sim 10^5 \frac{1}{\Omega \text{m}}$
  - $\kappa \sim 12 \text{W/m.K}$
  - Melting: $850 \text{ K}$.
How to operate?

Current:

\[ I = \frac{V}{r+R} \]

Voltage on load:

\[ V_R = I \cdot R = \frac{VR}{r+R} \]

Power at load:

\[ P = I^2 R = \frac{V^2 R}{(r+R)^2} \]

Maximum power:

\[ \Theta = \frac{\partial P}{\partial R} = \frac{V^2}{(r+R)^2} - \frac{V^2 R}{(r+R)^3} \cdot \frac{2}{2} \]

\[ = \frac{V^2}{(r+R)^3} \left( r+R - 2R \right) = 0 \]
\[ r = R \]

\[ \rho = \frac{1}{4} \left( \frac{V^2}{r} \right) = \frac{1}{4} \left( \frac{(S\Delta T)^2}{L/\sigma A} \right) \]

\( \sigma \) - conductivity

\( A \) - cross section area.

\( L \) - length.

\[ r = \frac{L}{\sigma A} \] - resistance of TE.
Negative Seebeck?


<table>
<thead>
<tr>
<th>Metals</th>
<th>Seebeck Coefficient</th>
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<td>-53</td>
</tr>
<tr>
<td>Bismuth</td>
<td>-72</td>
<td></td>
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</tbody>
</table>

Thoughts:

Imagine a gas of particles:

\[ E = n k_B T \]

\[ n = \frac{N}{V} = \text{density} \]
So for \( N = \text{constant} \):

\[ \dot{Q} \propto T \]

Particles move from high to low pressure.

Also, particles carry heat with them.

**Electrical current:**

\[ I = \frac{\delta A}{\delta t} \]

**Heat current:**

\[ \dot{Q} = k \frac{A}{L} \Delta T \]

**Particle current:**

\[ N \sim \frac{\dot{Q}}{e} \sim R \frac{A}{L} \frac{\Delta T}{k_B T} \]

\[ G = k_B T \sim \text{energy per particle} \]

Now:

\[ I = q \cdot \dot{N} \]

\( q \) - particle's charge.
So:
\[ \frac{A}{L} \cdot V = 9 \cdot \frac{1}{k_B T} \cdot \frac{kA}{L} \Delta T \]

And:
\[ \frac{V}{\Delta T} = \frac{k}{6} \frac{1}{k_B T} \]

\( k \), \( \delta \) always positive.

Sodbeck sign = sign \((9)\)

for holes:

\[ g = -e \]

(minus electron charge)

Also mentioned: Wiedemann-Franz Law

if \[ \frac{V}{\Delta T} \frac{k_B}{q} \sim \frac{k}{6} \frac{1}{k_B T} \]

then:
\[ \frac{G_T}{L^2} = \frac{q^2}{k_B^2} \cdot \frac{3}{4 \pi^2} \] (true in metals)
Meaning of "Couple"

Actual devices look like this:

Schematic of a Thermoelectric Cooler

Wires leading to load.

Effective circuit:

\[
\begin{align*}
S & \text{ gap} \\
\text{circuit} & \text{gap} \\
S & \text{ gap} \\
\text{load} &
\end{align*}
\]

(assume \( S = 15 \Omega \) = \( S \))
II. Pu heat production: \((^{238}_{92}PuO_2)\)

\[ Q = 500 \text{ W/kg} \]

Radiated heat:

\[ P = \sigma_\text{SB} A T^4 \]

Stefan-Boltzmann Constant

\[ \sigma_{SB} = 5.67 \times 10^{-8} \text{ W/m}^2 \text{ K}^4 \]

\[ \left[ \text{proof of SE superiority!} \right] \]
Assume a sphere of mass
\[ M = 0.5 \text{kg} \] (As turns out to be the case in Cassini)

radius:
\[ 0.5 \text{kg} = \frac{4}{3} \pi r^3 \cdot 10^{-4} \text{ kg/m}^3 \]

\[ r = \left(10^{-4} \cdot \frac{1}{8} \text{ m}^3\right)^{1/3} \]

\[ = \left(12 \cdot 10^{-6}\right)^{1/3} = 0.023 \text{ m} \]

\[ = 2.3 \text{ cm} \] (A bit bigger than our discussion in class)

Black body equilibrium

radiation coming out:

\[ \sigma A T^4 \]

radioactivity released heat

\[ \rho \frac{4 \pi r^3}{3} \]
Putting in $A = \pi r^2$:

$$\beta \frac{\pi}{3} r = 65B \cdot T_H$$

for $\text{H}_2O_2$

and:

$$T_H = \frac{\beta \frac{\pi}{3} r}{65B} = 10 \cdot \frac{500 \text{ W/kg} \cdot 0.023}{5 \cdot 6.7 \cdot 10^{-8} \frac{\text{W}}{\text{m}^2 \cdot \text{K}}}
\approx 900 \text{ K}$$

In practice: According to test publication, $T_H = 1100 \text{ K}$. Not exactly, but not bad.