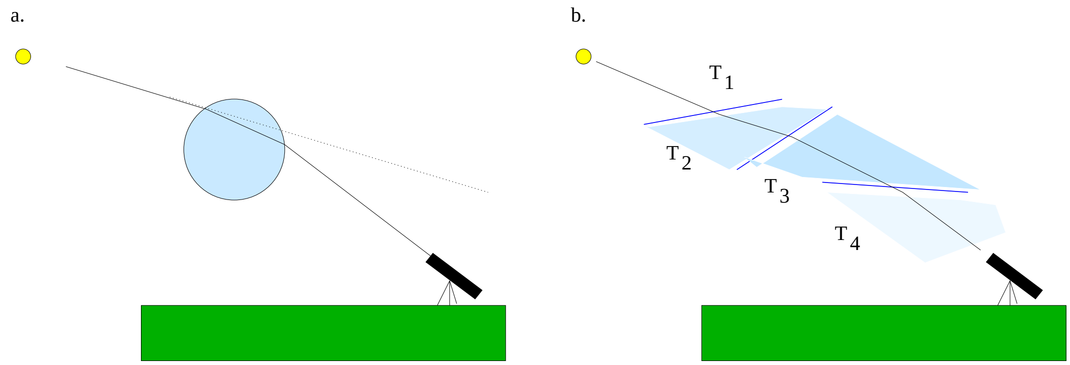

FIG. 1: An amateur astronomer is trying to observe a distant star. (a) Spherical cow model: the beam is distorted by a biggish spherical hot air region on its way, with the inside temperature different by about $1C$ from the outside. (b) Flat cow approximation: The beam goes through several different layers with temperatures fluctuating by order of $0.1C$ from one layer to the next. The extent of the layers (height, width, etc.) are of order $50m$ as well, we assume.

Allotted time: 2 hours

CPL- first winter challenge

Atmosphere vs. Astronomy

The atmosphere provides quite a headache to astronomers because of the distortions it produces in the light coming from stars and planets. Our goal in this problem is to estimate these distortions.

Consider an amateur astronomer trying to look through his 10' (0.25m) telescope at a far away planet at a 45^o to the horizon. The beam arriving at the telscope gets shifted by regions of temperature and pressure fluctuations which move through the beam's path at random. A rough formlula for the refractive index of air is:

$$n(T) \approx 1 + 3 \cdot 10^{-4}(1 - 0.0037T[C]) \tag{1}$$

where the temperature is supposed to be measured in centigrades (hence the [C]).

1. Find a formula connecting the angular shift $d\theta$ of a beam passing, through an interface of cold air with index n to warmer air with index $n - dn$, with an incidence angle relative to a normal to the surface of θ. Assume that both dn and $d\theta$ are tiny.

2. Using the spherical cow approximation, assume that all atmospheric fluctuations come as spherical clouds of extent of about $50m$, with the inside temperature of about 1C above or below the rest of the atmosphere (which is assumed uniform). *Estimate* the maximum angular shift that a cloud like that can cause to the beam which the telescope receives. In this model it is assumed that only one of these big thermals is in the way of the beam. (see Fig. 1a)

3. Shifting to the flat cow approximation, what if atmospheric fluctuations take the form of randomly oriented interfaces between region of temperature $T + \Delta T$, with ΔT of order $0.1C$ from layer to layer. What is the range (i.e., give an *estimate*) of the total angular shift in this case (if you wish, think of it as a variance)? Again assume that the relevant length scale for fluctuations is about $50m$. (see Fig. 1b)

How do the shifts you find compare, say, to the angular size of Jupiter perceived from Earth? (Distance $d \sim 780 \cdot 10^9 m$ and radius $142.8 \cdot 10^6 m$)