Magnetic pinball: solutions

We will take advantage of a constant of the motion that changes in a simple way at each impulse. In general, for a particle moving in a potential V, there is only one constant of the motion: the total energy (kinetic plus potential). For a free particle ($V = 0$), there are three: kinetic energy, x-momentum, and y-momentum. In this problem, with a particle in a magnetic field (and $V = 0$), the particle moves in a circle. Thus there are three constants of the motion:

1. x-position of center of orbit
2. y-position of center of orbit
3. radius of orbit

The radius of the orbit is proportional to the velocity:

$$r = \frac{m|\vec{v}|}{qB} \tag{1}$$

Let $u = \frac{m\vec{v}}{qB}$, $\vec{x}$ be the current position of the particle, $\vec{c}$ be the center of the orbit, and $R_{90}(\vec{v})$ be the 90-degree rotation of v clockwise around the origin. Then we have

$$\vec{c} = \vec{x} + R_{90}(\vec{u}) \tag{2}$$

(See figure.)

If we apply an impulse that sends $\vec{u} \to \vec{u} + \delta\vec{u}$, then $\vec{c} \to \vec{c} + R_{90}\delta\vec{u}$. Thus if we apply impulses $J\hat{y}$ in the y-direction, the center of the orbit remains at $y = 0$. Thus the best strategy is the one that increases the magnitude of radius $\vec{r} = \vec{x} - \vec{c}$ the most at each impulse. This occurs when $R_{90}(J\hat{y}) = J\hat{x}$ and $-\vec{r} = \vec{c} - \vec{x}$ are parallel.

1. Apply a pulse whenever the particle is moving upwards. After n pulses,

$$\vec{v} = \frac{nJ}{m}\hat{y} \tag{3}$$

$$\vec{u} = \frac{nJ}{qB}\hat{y} \tag{4}$$

$$y_{\text{max}} = |r| = |u| = \frac{nJ}{qB} \tag{5}$$

2. Apply the upward pulses when the particle is moving up, and the downward pulses when the particle is moving down.

3. It is possible to do slightly better than $y_{\text{max}} = h$. Apply a sequence of an even number of pulses so that the pinball scrapes the bottom ($y = -h$). Then when the ball's velocity is vertical, give it a (upward) pulse. At the top of this orbit, $y_{\text{max}} = h + \frac{J}{qB}$.

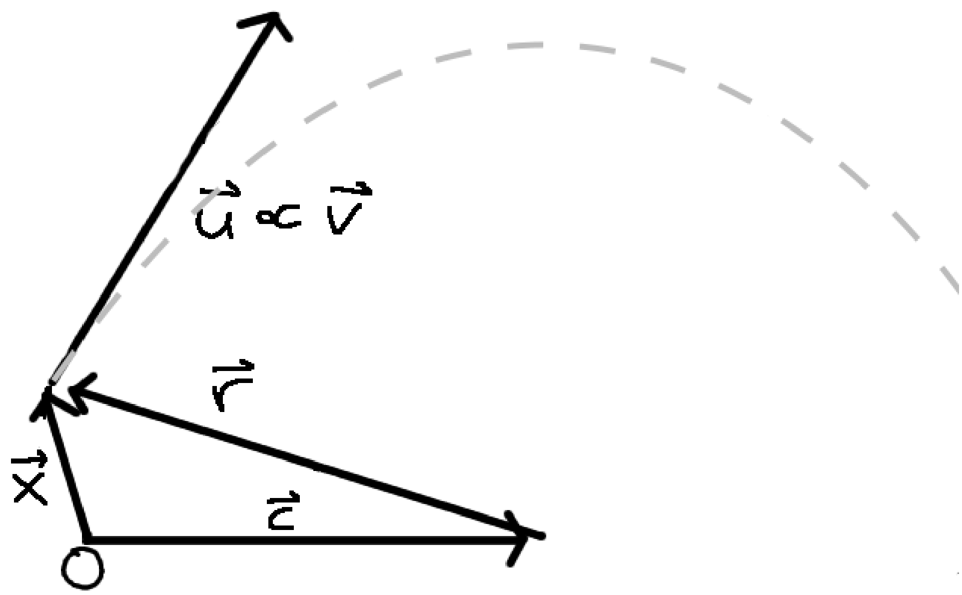

FIG. 1: The position of the particle is given by $\vec{x}$. Part of the circular orbit is shown in gray; the orbit has center $\vec{c}$. The center of the orbit is simply related to the velocity vector: $-\vec{r} = \vec{c} - \vec{x} = R_{90}(\vec{u})$.