



CPL - notes

Solar magnetic energy

The following idea was presented to me by Tal Carmon, a professor in University of Michigan. Tal claims that energy can be derived from the Earth's rotation using the solar magnetic field. The idea requires placing a superconducting loop of a radius of about $r = 1000km$ over Europe, roughly, such that the loop turns from parallel to normal and back every day. Let's assume that the wire cross section radius is $a = 0.01m$. Let's recap the meeting's discussion on the feasibility of this idea.

We would like to calculate the power produced by this wire under the best of conditions and most favorable assumptions. The power is:

$$P = V^2/r = I^2r = VI. \quad (1)$$

I. AVERAGE VOLTAGE

The first step is clearly to find the voltage available. This is done easily using Faraday Law:

$$\oint E \cdot dl = V = -\frac{d\Phi}{dt} \quad (2)$$

where Φ is the enclosed flux. Carmon quotes the magnetic field of the sun in our vicinity as $B_S = 6nT$ (please accept my MKS, SI, insistence ...). This makes it is easy to calculate the voltage:

$$V \sim \frac{\Phi_{max}}{t_{day}/2} = \frac{\pi \cdot (10^6m)^2 6 \cdot 10^{-9}T}{86400s/2} \approx 0.4V. \quad (3)$$

II. MAXIMUM CURRENT OF AN IDEAL SUPERCONDUCTOR

When we use a superconducting wire, there is no way to determine the resistance r of it. A superconductor has $r = 0$ linear resistance, and a rather complicated non-linear response to voltage. So using the V^2/r formula, which we would naturally do is out of the question. What we can do is determine the current through the superconductor.

A superconductor is like an inductor in the following sense. An inductor obeys $L \frac{dI}{dt} = V$, which we can also write as:

$$LI = N\Phi \quad (4)$$

where N is the number of loops in the inductor, and Φ the flux through each loop. A superconducting loop is like an inductor in the sense that the current through the superconductor is proportional to the flux that has crossed into the loop:

$$I = J\Phi \quad (5)$$

with J a constant with a name meant to honor Brian Josephson (check out http://en.wikipedia.org/wiki/Brian_David_Josephson_for_an_interesting_reading.Makes_sure_to_scroll_down_to_parapsychology). Now we

This is a classical case of having to guess something without knowing anything about the theory. To get a handle on things though, we must think as hard as possible of equations that give such a behavior. Here the hint is that inductive elements resemble motion of free particles. $L \frac{dI}{dt} = V$ is essentially like $m \frac{dv}{dt} = F$ since current is proportional to velocity. This makes it possible to just look at the Schrödinger equation (I know that this is quite a jump). In the SE:

$$E\psi = \frac{1}{2m} \left(\frac{\hbar}{i} \nabla - e\vec{A} \right)^2 \psi \quad (6)$$

Notice that this is the SE in the presence of a magnetic field, $\vec{B} = \nabla \times \vec{A}$. But essentially what the SE is, represents simply the kinetic energy of a particle: $\frac{1}{2}m\vec{v}^2$. This seems to indicate that:

$$\vec{v} \rightarrow \frac{1}{m} \left(\frac{\hbar}{i} \nabla - e\vec{A} \right) \quad (7)$$

so the velocity is not just the momentum operator over m . Rather, we must take off the *vector potential contribution*. This is the origin of the Aharonov-Bohm effect. Regardless of name dropping, this tells us that if we would like to construct the current operator, we should not forget \vec{A} . The standard current operator *without* magnetic field is:

$$\vec{j} = \frac{1}{m} e \operatorname{Re} \psi^* \frac{\hbar}{i} \nabla \psi \quad (8)$$

Here it is obvious how to modify this to get the magnetic field involved:

$$\frac{\hbar}{i} \nabla \rightarrow \frac{\hbar}{i} \nabla - e\vec{A} \Rightarrow \vec{j} = \frac{1}{m} e \operatorname{Re} \psi^* \left(\frac{\hbar}{i} \nabla - e\vec{A} \right) \psi. \quad (9)$$

okay! What is a superconductor? If we don't know anything about it, then we can just guess that a superconductor is where all electrons can move without resistance. Ignoring the gradient, we see that:

$$\vec{j} \rightarrow \frac{e^2}{m_e} \vec{A} |\psi|^2. \quad (10)$$

Now that $|\psi|^2$ is the probability density of the one-electron wave function. Sum up the entire contribution of all electrons, and what do you get? $\sum_n |psi_n|^2 \sim N|\psi|^2 = n_S$ the (superconducting) electron density. So we get:

$$\vec{j} = \frac{e^2}{m_e} \vec{A} n_S. \quad (11)$$

As it turns out that it is *Cooper pairs* which consist of two electrons, that are the objects which can move without resistance. But it doesn't matter for the current formula - once we change the charge mass and density to reflect charge $2e$ particles with mass $2m_e$ all factors of 2 cancel.

Eq. (11) is the London equation, named after Heinz and Fritz London. It was a phenomenological guess that gave us the first tool for understanding the behavior of superconductors. In our case, it gives us a tool to estimate the current in the loop.

First, we need to determine the vector potential:

$$\oint \vec{A} \cdot d\vec{\ell} = \Phi \rightarrow 2\pi R |\vec{A}| = B\pi R^2 \quad (12)$$

so:

$$I = \pi a^2 j = \pi a^2 \frac{e^2}{m_e} n_S \frac{BR}{2} \quad (13)$$

putting in numbers:

$$I = \pi 10^{-4} m^2 \frac{2.6 \cdot 10^{-38}}{10^{-30} kg} \frac{1}{10^{-28} m^3} \frac{6 \cdot 10^{-9} \cdot 10^6 m}{2} \approx 2 \cdot 10^{14} A \quad (14)$$

I have to admit that the first time I calculated this I made an arithmetic error, and got $10^6 A$. Ah well. This current, i.e., the real answer above, is way too big. It has the risk of frying the wire. So we need to make sure that we didn't over calculate.

III. MAXIMUM CURRENT A SUPERCONDUCTOR CAN TAKE

What other principle can limit the current? Here one needs some knowledge of superconductors. They cannot take too much magnetic field. As can be found by surfing the web (an example is posted on the website), high T_c superconductors like YBCO can take about $B_c \sim 1T - 10T$. For a wire of radius of $a = 1cm$, how much current would produce $1T$ on the rim of the wire?

The magnetic field obeys Ampere's law:

$$\oint \vec{B} \cdot d\vec{\ell} = \mu_0 I_{enclosed} \quad (15)$$

So:

$$I_c = \frac{B_c 2\pi a}{\mu_0} \sim \frac{2\pi(1T - 10T)0.01m}{4\pi 10^{-7}} \approx 0.510^5 A - 0.5 \cdot 10^6 A \quad (16)$$

Indeed this range is orders of magnitude smaller than the current we calculated an ideal superconductor with $B_c \rightarrow \infty$ would be able to give. Note that in the paper I found discussing the maximum field on a wire, there is also a measurement of the critical current, which indeed is about $10^6 A/cm^2$ (just look at the graphs - don't bother with the text...).

IV. MAXIMUM POWER

Now we can calculate the power. Giving the full benefit of the doubt, we take the maximum value of the current and multiply it by the average voltage:

$$P_{max} = I_{max} V = 0.4V \cdot (0.5 \cdot 10^6 A) \rightarrow 0.2 \cdot 10^6 W = 0.2 MW. \quad (17)$$

Not bad! But doesn't look like it is worth the effort. Carmon estimates this to be $54 MW$. We find a result which is two orders of magnitude short.