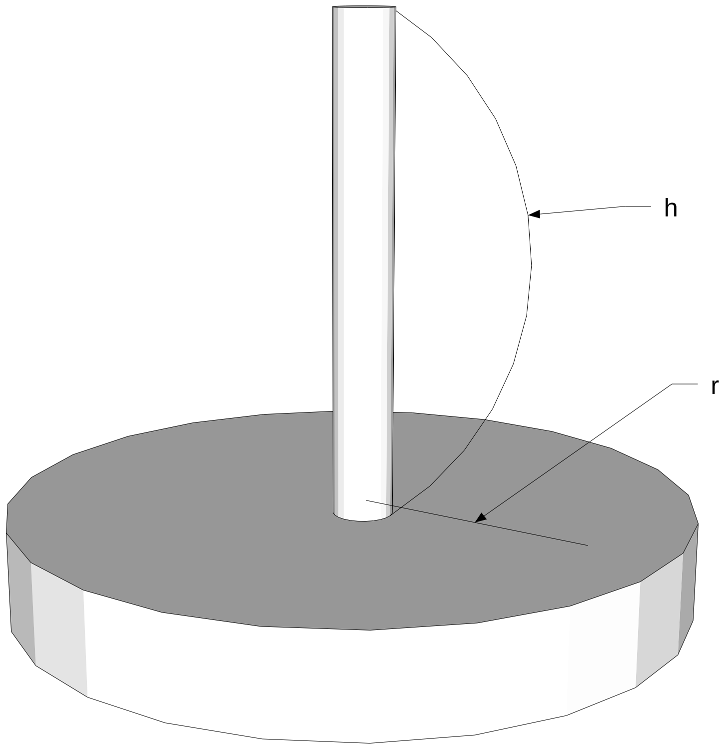
CPL Solar Tower note(by Gil Refael and Isaac Kim)



I. POWER ESTIMATION FOR DESERT CHIMNEY

In this note, we will study the so called 'desert chimney' as a new way to harvest energy from sun. Idea is simple. We use the energy from sun to heat up air. This process will increase the kinetic energy of the gas. As these heated gas is emitted from the chimney, it rotates the turbine.

Let us first start with a simple model. Conservation of energy results in a conservation of the following quantity.

$$\frac{1}{2}\rho v^2 + \rho g h + p + \rho C_V T \tag{1}$$

This is basically a slight generalization of Bernoulli's principle, although you can easily see where the last term came from. When we derive Bernoulli's principle, we typically do not care about heat energy, whereas here we do.

We shall make a linear approximation that pressure difference between the bottom and top of the chimney is $\rho_0 g h$, where ρ_0 is the air density at outside temperature. Inside the chimney, however, has a different temperature, and hence different density.

$$\frac{1}{2}\rho_T v^2 = (\rho_0 - \rho_T) g h. \tag{2}$$

Therefore, $v^2 = 2(\frac{\rho_0}{\rho_T} - 1) g h \approx 2(\frac{T}{T_0} - 1) g h$.

Now let's put in some numbers. For outside temperature $T_0 \approx 273K$ and inside temperature $T \approx 373K$, we have

$$v = \sqrt{\frac{2}{3} g h} = \sqrt{\frac{2}{3} 10 \times 10^3} \approx 80 m/s. \tag{3}$$

In class we derived an expression of power in terms of the velocity, turbine cross section area, and density of the fluid. For detailed analysis, read Gil's note on tidal energy.

$$P = \frac{1}{6}\rho A v^3 \approx 150MW, \tag{4}$$

for cross section area of $3m^2$ and air density at temperature $T \approx 300K$.

II. ESTIMATION OF THE INSIDE/OUTSIDE TEMPERATURE

Power generated from sun is

$$P = A\sigma T^4 \tag{5}$$

When this energy reaches earth, the power per area becomes

$$\frac{P}{A} = \frac{R_S^2}{R_{SE}^2}\sigma T^4, \tag{6}$$

where R_S is the radius of the sun and R_{SE} is the distance between sun and earth. $R_S \approx 0.7 \times 10^9 m$ and $R_{SE} \approx 150 \times 10^9 m$,which gives $\frac{P}{A} \approx 1300W/m^2$. Suppose that all the energy comes into the gas. This energy will be used in doing work and heating up the gas. In an infinitesimal form,

$$C_P \frac{dT}{dt}\Delta N = 2\pi r \Delta r \frac{dP}{dA}, \tag{7}$$

Using $PV = NkT$, we have $\Delta N = \frac{P\Delta V}{kT} = \frac{2\pi r \Delta r h P}{kT}$. To calculate the velocity at each radius r, we use the following relation.

$$\frac{dT}{dr} = \frac{dT}{dt}\frac{1}{v(r)}. \tag{8}$$

For each infinitesimal segment, the amount incoming gas must be equal to the amount of outgoing gas, which gives us the following relation.

$$v(r) 2\pi r h \rho(r) = const \equiv L \tag{9}$$

Using ideal gas law, we have

$$v(r) = \frac{kT}{mP}\frac{L}{2\pi r h}, \tag{10}$$

where m is the average mass of the air molecule. Combining all these together, we have

$$\frac{7}{2}\frac{k}{m}\frac{dT}{dr}\frac{1}{2\pi r}L = \frac{dP}{dA}. \tag{11}$$

First let's estimate the constant L.

$$L = A_c v_c \rho_{top} \approx A_c v_c \frac{P}{kT_0}, \tag{12}$$

where A_c is the cross section of the chimney, v_c is the speed of gas at the top, and ρ_{top} is the density at the top. Since we calculated $v_c^2 = 2gh(\frac{T}{T_0} - 1)$ in the previous section, it is tempting to use it again! But let's start thinking from the first principle.

$$\frac{1}{2}\rho v^2 = \rho g h, \tag{13}$$

but really Bernoulli's principle should come from energy conservation.

$$(\rho_{out} - \rho_{in}) g H = \frac{1}{2} v^2 \rho_{in}. \tag{14}$$

Assuming P is constant, we have

$$\frac{\rho_{out}-\rho_{in}}{\rho_{in}}=\frac{T_{in}}{T_{out}}-1=\frac{1}{2}\frac{v^2}{gH}. \tag{15}$$

Now we can finally calculate the power.

$$\begin{aligned}
P &= \frac{1}{3}\rho v^3 A && (16)\\
&= \frac{1}{3}Lv^2 && (17)\\
&= \frac{1}{3}\frac{m\pi R_0^2}{\frac{7}{2}k(T_{in}-T_{out})}\frac{dP}{dA}2gH\frac{T_{in}-T_{out}}{T_{out}} && (18)\\
&= \frac{4}{21}\frac{mgH}{kT_{out}}\pi R_0^2\frac{dP}{dA} && (19)
\end{aligned}$$

The actual amount of energy that the tower receives is $\pi r^2\frac{dP}{dA}\approx 4000MW$. Therefore the efficiency is

$$\frac{P}{P_{input}}=\frac{4}{21}\frac{mgH}{kT_{out}}\approx\frac{4}{21}\frac{30\times 1.66\times 10^{-27}\times 10\times 1000}{1.38\times 10^{-23}\times 300}\approx 2.2\times 10^{-2}. \tag{20}$$