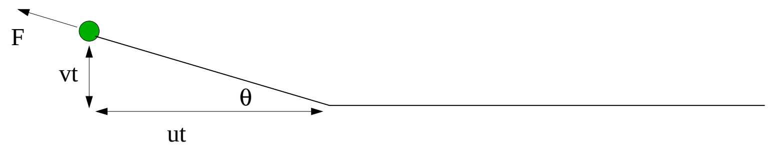

FIG. 1: We examine wave propagation in a taut wire by pulling on it up on one side and seeing how the disruption propagates. The edge of the rope moves up with velocity v, and the disruption moves with velocity u, which is the wave velocity.

Waves in ropes

In the first meeting we discussed an unconventional way for finding out the speed of wave motion in a taut rope. The steps were important not only for understanding the speed of wave propagation, but also how the rope acts on whatever holds it on its tips.

We start with a rope with tension T and linear density ρ. It is being pulled at its edge with a force F, such that the tip also moves with velocity v (see figure). The disruption propagates in the rope with speed u.

First we note that the angle θ in the figure is:

$$\tan\theta \approx \theta = \frac{v}{u}. \tag{1}$$

Where we also assume that $v \ll u$ and that the angle is small. By thinking about the free-body diagram at the tip of the rope we find that:

$$F_x = T\cos\theta \approx T \tag{2}$$

and, more importantly:

$$F_y = T\sin\theta \approx T\theta \approx \frac{T}{u}v \tag{3}$$

So the force on the tip looks like a dissipation term.

Now, where is this force F_y applied to? It is applied to put momentum into the moving segment of the rope. The size of the moving segment is ut, where t is time, and its velocity is v. Therefore its momentum as a function of time is:

$$P = \rho v(ut) \tag{4}$$

and by balancing the force:

$$F_y = \frac{dP}{dt} = \rho uv \tag{5}$$

Again the same dissipative form. Equating Eqs. (3) and (5) gives:

$$\rho uv = \frac{T}{u}v \tag{6}$$

and finally:

$$u = \sqrt{\frac{T}{\rho}}. \tag{7}$$