Ph 129 Mid-term

1. Dissipative string. Consider the modified wave equation for the displacement of a string:

$$\frac{\partial^2 \psi}{\partial t^2} + 2\lambda \frac{\partial \psi}{\partial t} + \lambda^2 \psi - \frac{\partial^2 \psi}{\partial x^2} = 0. \tag{1}$$

with $\lambda > 0$.

(a) What is the Green function, $G(x, t)$ of this equation?

(b) The string is made to oscillate by applying a concentrated force $F(t) = \cos \omega t$ at $x = 0$:

$$\frac{\partial^2 \psi}{\partial t^2} + 2\lambda \frac{\partial \psi}{\partial t} + \lambda^2 \psi - \frac{\partial^2 \psi}{\partial x^2} = \delta(x) \cos(\omega t) \tag{2}$$

What is the motion of the string $\psi(x, t)$?

2. Off-center cylindrical capacitor.

(a) Consider the conformal mapping,

$$z = \frac{w - w_0}{\bar{w}_0 w - 1}, \tag{3}$$

where w_0 is an arbitrary complex number. Show that the conformal mapping (3) maps the unit circle centered at the origin to itself.

(b) Consider two infinite parallel conducting cylinders;

$$\begin{array}{l} \text{cylinder 1: radius } r_1 = 1, \quad \text{electric potential } V_1 = 0 \\ \text{cylinder 2: radius } r_2 = 2/5, \quad \text{electric potential } V_2 = 1. \end{array} \tag{4}$$

Cylinder 2 is inside cylinder 1, such that distance between the centers of cylinders 2 and 1 is 2/5. Using the conformal mapping method find the electric potential in the space between the cylinders.

Hint: w_0 could be chosen real without loss of generality. Conformal mapping (3) can be used to map the two-cylinder configuration to simple cilyndrically symmetric geometry.

3. Moving frame Schrödinger equation. Find the (causal) Green function of the following equation:

$$i\frac{\partial \psi}{\partial t} + \frac{1}{2m}\frac{\partial^2 \psi}{\partial k^2} - v\frac{\partial \psi}{\partial k} = 0 \tag{5}$$

with m, and v some constants.

4. Inverse scattering from an uneven dipole. What is the long-time solution of the KdV equation:

$$\frac{\partial u}{\partial t} - 6u\frac{\partial u}{\partial x} + \frac{\partial^3 u}{\partial x^3} = 0 \tag{6}$$

with initial condition:

$$u(x, t = 0) = -V_1 \delta(x) + V_2 \delta(x - x_0) \tag{7}$$

with V_1, V_2 both positive, and $V_2 < V_1$? Assume that $0 < x_0 V_1 \ll 1$, concentrate only on the soliton part of the answer (neglect the incoherent non-soliton part of the answer). Also ignore the fact that the structure of the potential is going to shift the soliton initial point (no need to calculate the shift x_n).