

This examination is open book. You may consult your textbook and class notes, your own graded problem sets, and the solutions to the homework posted on the web. No other written material may be used. No computer algebra or similar programs may be used.

You have **4 hours** in which to work, by yourself, on this exam. You must do the exam in one sitting, although you may take up to 30 minutes of additional break time.

Do not consult with anyone about this exam until after 5:00 pm, Wednesday, March 18.

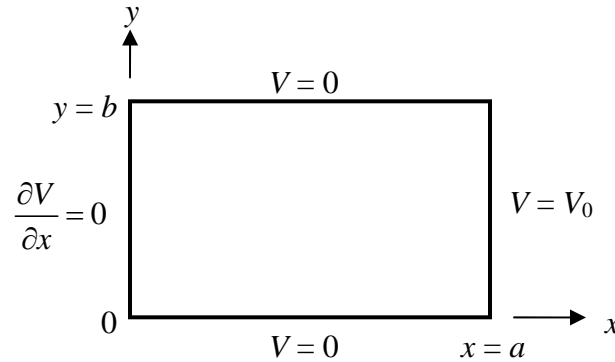
Be sure to sign your name on each page of your exam.

The exam is due by 5:00 pm on Wednesday, March 18. Please turn it in to Loly Ekmekjian in 114 Sloan Annex.

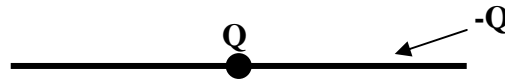
**Do not proceed to the next page until you are ready for
the 4 hour clock to begin!**

GOOD LUCK!

Problem 1 (25 points) Find the electric potential $V(x,y)$ everywhere inside a long conducting pipe of rectangular cross-section (defined by $0 < x < a$ and $0 < y < b$) subject to these boundary conditions:



Problem 2 (25 points) A point charge $+Q$ lies at the center of a charged rod of length L . The rod carries an equal but opposite charge $-Q$ distributed uniformly along its length. Choose a sensible coordinate system and make a Legendre expansion of the electrostatic potential $V(r)$, valid at distances r from the central charge that are larger than $L/2$. Evaluate the coefficients of the first two non-vanishing terms explicitly.



You may find this helpful: $\ln(1+x) = x - x^2/2 + x^3/3 - x^4/4 + \dots$

Problem 3 (25 points)

a) (10 points) In class we studied the so-called “physical dipole”; two charges, $+q$ and $-q$ separated by a distance d along the z -axis. We found that to leading order the potential $V(r,\theta)$ viewed from far away was approximately $V(r,\theta) = \frac{p \cos \theta}{4\pi\epsilon_0 r^2}$ with $p = qd$ being the dipole moment. We noted that if we allowed $q \rightarrow \infty$ while $d \rightarrow 0$ in such a way that $qd = p$ remains constant, the simple dipole potential becomes exact (i.e. all higher order multipoles vanish).

Suppose now that you were solving this problem again, with the only difference being that the two charges are immersed in an infinite linear dielectric medium with dielectric constant ϵ . Show that the potential far away is still that of an ideal dipole, except with a different total moment p' . Express p' concisely in terms of p and ϵ .

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Problem 3, continued:

b) (15 points) In class we also showed that the electric potential $V(r,\theta)$ outside of a uniformly polarized sphere of radius R is given exactly by the ideal dipole formula:

$$V(r > R, \theta) = \frac{p \cos \theta}{4\pi\epsilon_0 r^2}. \text{ In this result } p = \frac{4}{3}\pi R^3 P \text{ is the total dipole moment of the}$$

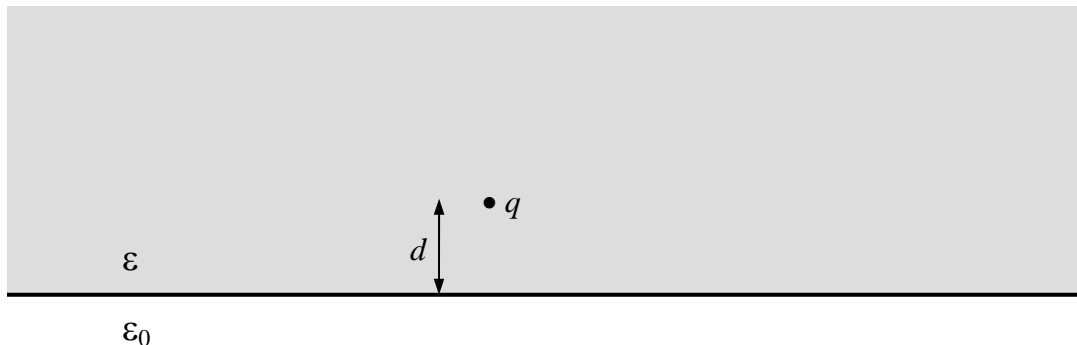
sphere and P is the polarization (here taken to lie along the positive z -axis).

Now, in analogy to part a), imagine that the same polarized sphere is surrounded by a linear dielectric medium, with dielectric constant ϵ . Show that the potential outside the sphere is still that of an ideal dipole, except with a different total moment p' . Express p' concisely in terms of p and ϵ .

Hint: Do not be alarmed if the potential in parts **a)** and **b)** differ (even when the two dipole moments $p = qd$ and $p = \frac{4}{3}\pi R^3 P$ are equal). This peculiar fact, which remains true even if both objects are shrunk down to ideal point dipoles, apparently went unnoticed until 2002!

Problem 4 (25 points)

A point charge q is immersed in a semi-infinite dielectric half-space, a distance d from the interface of the dielectric and vacuum. The dielectric is linear and has dielectric constant $\epsilon > \epsilon_0$.



a) (10 points) Is the charge attracted or repelled from the interface?

b) (15 points) What is the magnitude of the force experienced by the point charge?