Physics 106c **Final Examination** **06/02/09**

This examination is open book. You may consult your class notes, your own graded problem sets, and the solutions to the homework posted on the web. No other written material may be used. No computer algebra or similar programs may be used.

You have **4 hours** in which to work, by yourself, on this exam. You must do the exam in one sitting, although a couple of short (~10 minute) breaks are OK.

There are 5 problems, worth a total of 75 points.

Do not consult with anyone about this exam until after 5:00 pm, Friday, June 12.

Be sure to sign your name on each page of your exam.

EXAM DUE DATE/TIME:

Seniors and graduate students: 5:00 pm, Friday, June 5.

Undergraduates: 5:00 pm, Friday, June 12.

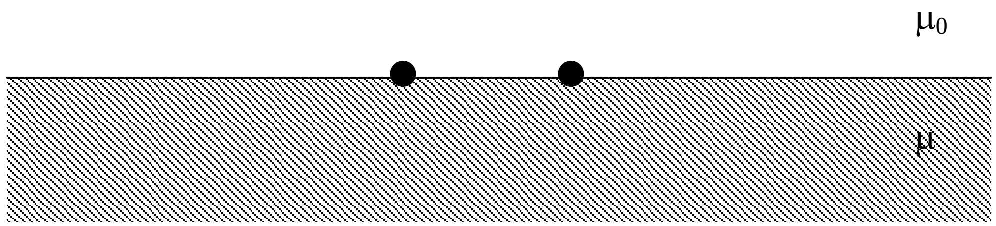
Please turn it in to Loly Ekmekjian in 114 Sloan Annex.

As always, you are bound by the Honor Code in all matters concerning this exam.

Do not proceed to the next page until you are ready for the 4 hour clock to begin!

GOOD LUCK!

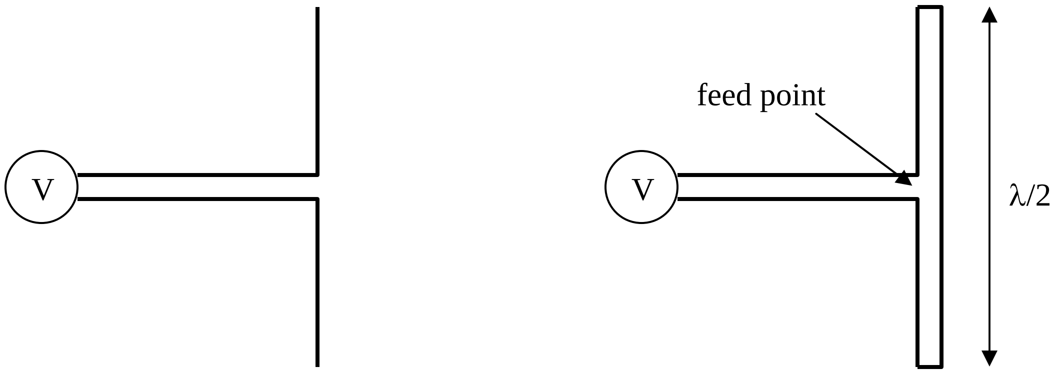
Problem 1 (5 points)
Two long parallel wires are fixed onto the surface of a semi-infinite material of magnetic permeability μ. The wires are insulated from the material and carry currents I_1 and I_2, respectively. The separation between the wires is d and their radii are negligibly small. Find the force between the wires, per unit length. In cross-section:



Problem 2 (10 points)
A copper wire of diameter 1 mm is carrying a current of 1 milliamp amplitude at 10 MHz. Estimate the time averaged amount of heat generated, per meter of length, in the wire. The resistivity of copper is 1.6×10^{-8} ohm-meter.

Problem 3 (15 points).
a) (5 points) A particularly simple transmission line consists of two parallel wires. Taking the radius of each wire to be a and the distance between the wire centers to be d, find an approximate formula for the velocity of propagation and characteristic impedance of this transmission line. You may assume $d >> a$ and that the wires are in vacuum.

This transmission line can be connected to a "center-fed half-wave dipole" antenna as we discussed in class (and shown below on the left) or to a "double hair-pin" loop antenna shown on the right. The other end of the line is connected to a voltage source at frequency ω. In both cases the length of the antenna is $\lambda/2$. The spacing d of the wires in the transmission line and in the hair-pin loop is much less than λ.



b) (10 points) Assuming that the current injected into the two antennas at the feed point is the same, determine the ratio of the total power radiated in the two cases. Recalling that the radiation resistance, or impedance, of the center-fed half wave dipole is approximately 73 ohms, determine the corresponding radiation resistance of the hair-pin loop.

Problem 4 (20 points)

A particle of charge q and mass m is heading in from infinity toward a fixed charge Q. The two charges have the same sign and the trajectory is exactly head-on. Obviously, q slows down, turns around, and heads back to infinity. The initial kinetic energy of charge q is $\frac{1}{2} m v_i^2$, and $v_i \ll c$.

Owing to its deceleration and subsequent re-acceleration, the charge q radiates electromagnetic waves. Thus the velocity v_f of q when it returns to infinity will be less than v_i. Your job is to estimate the difference, assuming the effect is very small.

a) 5 points **-** Ignoring the radiation entirely, find a formula for the acceleration a of the charge q as a function of its instantaneous velocity v and initial velocity v_i.

b) 15 points **–** Calculate the total energy lost to radiation. Show that the fractional change in the kinetic energy over the entire trip is indeed small, given that $v_i \ll c$.

Problem 5 (25 points)

An ideal coax having characteristic impedance R_c is terminated by an ideal inductor L_0. In other words, the inductor connects the inner to the outer conductor of the coax at one end. For definiteness, let the coax lie along the x-axis, extending from $x < 0$ to $x = 0$ where the inductor is attached.

a) 5 points **-** A sinusoidal wave $exp[i(-kx + \omega t)]$ wave is incident upon the inductor from the left ($x < 0$), and a reflected wave $R\, exp[i(kx + \omega t)]$ is generated. Find the reflection coefficient R.

Now assume the coax has a finite length, running from $x = -d$ to $x = 0$ where the inductor is. A ideal voltage source is attached to the coax at $x = -d$. This voltage source maintains the voltage at this location equal to $V_0\, exp(i\omega t)$.

b) 5 points **–** Writing the voltage wave as $V(x,t) = A\, exp[i(-kx + \omega t)] + B\, exp[i(kx + \omega t)]$, solve for A and B in terms of the various parameters of the problem.

c) 10 points **-** Find the impedance Z that this coax + inductor presents to the voltage source. (Z is defined as the ratio of the voltage $V(x,t)$ to the current $I(x,t)$ supplied by the voltage source.) Express Z as compactly as possible in terms of R_c, k, d, ω and L_0. Prove that Z is pure imaginary.

d) 5 points **-** Show that Z diverges at certain frequencies. Specify the conditions for this divergence. Assuming that the coax is short, i.e. $kd \ll 1$ (but not zero), show that the lowest frequency divergence occurs when $\omega^2 = 1/L_0 C$, with C the *total* capacitance of the coax.