

This examination is open book. You may consult your textbook and class notes, your own graded problem sets, and the solutions to the homework posted on the web. No other written material may be used. No computer algebra or similar programs may be used.

You have **4 hours** in which to work, by yourself, on this exam. You must do the exam in one sitting, although you may take up to 30 minutes of additional break time.

Do not consult with anyone about this exam until after 5:00 pm, Tuesday, May 5.

Be sure to sign your name on each page of your exam.

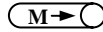
The exam is due by 5:00 pm on Tuesday, May 5. Please turn it in to Loly Ekmekjian in 114 Sloan Annex.

Do not proceed to the next page until you are ready for the 4 hour clock to begin!

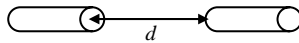
GOOD LUCK!

Problem 1 (10 points)

a) 4 points - Find the magnetic fields \mathbf{B} and \mathbf{H} everywhere along the symmetry axis of a uniformly magnetized cylindrical rod of radius a and length L . The magnetization \mathbf{M} of the rod points along its symmetry axis. Make a sketch of \mathbf{B} and \mathbf{H} all along the symmetry axis.



b) 4 points - Now suppose there is a second identical rod, coaxial with the first, and with its \mathbf{M} pointed in the same direction. Let the distance between the facing ends of the two rods be d . Assume $d \gg a$. What is the force between these two bar magnets?



c) 3 points - For $d \gg L$ the force between the rods should fall off as d^n . What is n ?
Hint: You may find this integral helpful:

$$\int \frac{dx}{(c^2 + (x-b)^2)^{3/2}} = \frac{x-b}{c^2 \sqrt{c^2 + (x-b)^2}}$$

Problem 2 (10 points)

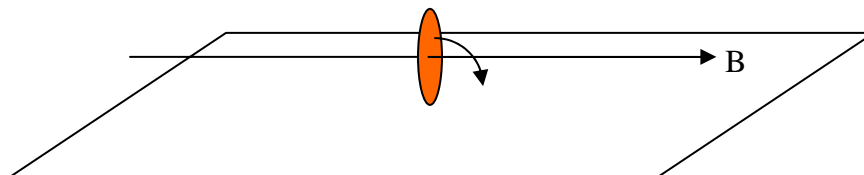
A copper penny is balanced on edge on a flat surface. A uniform magnetic field $B=2$ T points perpendicular to the face of the penny. The top edge of the penny is given a very slight nudge. The penny falls over, face down onto the flat surface. How long does it take?

For reference, pennies are 1 cm in radius and 1.5 mm thick. Their mass is about 3 grams. The electrical resistivity of copper is 1.6×10^{-8} ohm-meter. (This is a 50 year-old penny, from back when they really were made of copper.)

You do not need to find an exact answer to this problem, anything within a factor of ~ 2 or so is good enough. You must, however, arrive at the correct dependence of the fall time on all the relevant physical parameters: penny dimensions and mass, magnetic field, acceleration due to gravity, and copper resistivity. Justify any approximations you make.

You may assume that the penny does not slip or roll, it just flops over.

(In case you are interested, modern pennies are mostly zinc. The resistivity of zinc is about 4 times larger than copper.)



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Problem 3 (5 points)

A distribution of permanent magnetization, $\mathbf{M}(\mathbf{r})$, exists in a localized region of space. There are no other sources of magnetic field anywhere. For such a system, the total stored energy may be written as

$$U = \frac{\mu_0}{2} \int_{\text{all space}} |\mathbf{H}|^2 d\tau$$

Prove that, for this system, the above result is equivalent to:

$$U = -\frac{\mu_0}{2} \int_{\text{all space}} \mathbf{M} \cdot \mathbf{H} d\tau$$

Problem 4 (15 points)

A linearly polarized electromagnetic wave of wavevector k_0 is normally incident upon a metal film of thickness t and conductivity σ . Find the electric fields associated with the transmitted and reflected beams, in units of the incident electric field. Assume the medium on both sides of the metal film is vacuum and that the permeability and dielectric constant of the metal are simply μ_0 and ϵ_0 , respectively. Express your answers in terms of the complex wavevector \tilde{k} of the metal and the (real) vacuum wavevector k_0 and the film thickness. You DO NOT need to rationalize the final results.

Prove that your results make sense in the limits that t is large and small in comparison to the skin depth of the metal.

You may recall from a previous course that for a *dielectric* thin film, the reflection and transmitted field show resonant behavior. Do such resonances occur in the present case? Justify your answer.

