Physics 106c **Final Examination** **05/30/08**

This examination is closed book. You may consult your class notes, your own graded problem sets, and the solutions to the homework posted on the web. No other written material may be used. No computer algebra or similar programs may be used.

You have **4 hours** in which to work, by yourself, on this exam. You must do the exam in one sitting, although a couple of short (~10 minute) breaks are OK.

Do not consult with anyone about this exam until after 5:00 pm, Friday, June 13.

Be sure to sign your name on each page of your exam.

EXAM DUE DATE/TIME:

Seniors and graduate students: 5:00 pm, Friday, June 6.

Undergraduates: 5:00 pm, Friday, June 13.

Please turn it in to Loly Ekmekjian in 114 Sloan Annex.

As always, you are bound by the Honor Code in all matters concerning this exam.

Good luck, and have a great summer!

Problem 1 (20 points)

Two long coaxial cables are joined at one point. Let the wave velocity and characteristic impedance for coax #1 be v_1 and R_{c1} and for coax #2, v_2 and R_{c2} . Assume a voltage wave $f(x-v_1t)$ is moving down coax #1, heading toward the junction with coax #2.

a) (5 points) What is the current wave accompanying the voltage wave $f(x-v_1t)$?

b) (10 points) Find the reflected and transmitted voltage and current waves.

c) (5 points) Find the amplitude of the reflected voltage wave, relative to the incident, for the specific case of R_{c1} = 50 ohms, R_{c2} = 75 ohms.

Problem 2 (20 points)

a) (10 points) Design a waveguide (with rectangular cross-section) in which the cut-off frequencies of the lowest two propagating modes are 10 GHz and 18 GHz respectively. Give real physical dimensions for the cross-section.

b) (5 points) Are these modes TE or TM?

c) (5 points) How many modes of each type propagate in the band from 18 to 22GHz?

Problem 3 (25 points)

A linearly polarized plane electromagnetic wave of frequency ω, propagating in vacuum along the z–axis with an electric field of amplitude E_0 polarized along the x–axis, falls upon a point charge q of mass m, originally sitting at the origin of coordinates. The point charge is free to move in response to the electromagnetic wave.

a) (7 points) Solve for the motion of the charge, assuming that its velocity remains very small compared to that of light.

b) (8 points) The moving charge radiates its own electromagnetic waves. Find magnitude and angular dependence of the time-averaged outgoing power, in terms of E_0, q, m, and ω. Be clear about any angles you need to define.

c) (10 points) The total scattering cross-section is defined to be that area σ over which the total power in the *incoming* electromagnetic wave equals the total power (integrated over all solid angle) radiated by the point charge. Find a formula for σ appropriate to the preceding discussion.

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Problem 4 **(35 points)**

A linearly polarized plane electromagnetic wave of frequency ω, propagating in vacuum along the *z*–axis with an electric field of amplitude E_0 polarized along the *x*–axis, is normally incident upon a very thin metal sheet. The metal sheet lies in the x-y plane. Assume that the penetration depth δ at the frequency ω is much greater than the sheet thickness *t*.

For parts a), b), and c) assume that the metal sheet has an ordinary ohmic conductivity σ.

a) (8 points) Find the reflected and transmitted electromagnetic waves (both the electric and magnetic fields). You may assume that the sheet is essentially infinitely thin, but that the product $\sigma\, t$ is non-zero.

b) (7 points) Find the fraction of the incoming energy flux which is reflected, transmitted, and absorbed by the metal sheet.

c) (5 points) Determine what conditions (on σ and *t*) will maximize the energy absorption. What is the maximum energy absorption fraction?

Now we will assume that the conductivity of this particular metal is somewhat unusual. We shall replace the usual relation between current density and electric field, $\boldsymbol{J} = \sigma\, \boldsymbol{E}$ with the following:

$$K_x = \sigma_H E_y$$
$$K_y = -\sigma_H E_x$$

In other words, the *sheet current density* $\boldsymbol{K}$ in the thin metal plate is related to the electric field $\boldsymbol{E}$ via a new conductivity coefficient, σ_H. Note that the x-component of the current density is determined by the y-component of the electric field, and the y-component of the current density by the x-component of the electric field. Note also the minus sign. While this situation may seem peculiar to you, it is in fact readily encountered when certain metals are subjected to intense static magnetic fields. Do not worry about how this situation comes about; just assume it is true.

d) (7 points) Once again, find the reflected and transmitted electromagnetic waves. Be careful about what you assume concerning the polarization of the outgoing waves. (The incident wave is linearly polarized as before.)

e) (4 points) What fraction of the incoming power is absorbed?

f) (4 points) Describe, quantitatively, the polarization state of the transmitted beam.