

Physics 106c: Electrodynamics

Problem Set 2

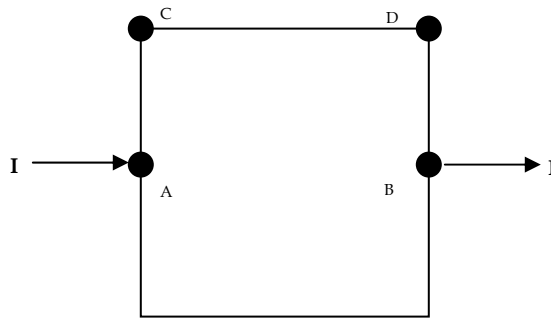
Due: 4pm, Friday, April 17, 2009

Remember: Late homework will be granted 50% credit up to one week late, unless you have a note from the Dean or a health official.

Reading: Griffiths Chapters 6 and 7

Problems:

In class we solved for the current distribution in a thin square conducting sheet in which current I was injected at point A and withdrawn at point B. We determined the voltage difference V_{cd} between two corner contacts. The geometry appeared thus:



1. Generalize our results to a rectangular sheet of width a and length b (with b being the distance between contacts c and d). Evaluate numerically the resistance R (defined as the ratio of the voltage V_{cd} to the current I) for enough values of the aspect ratio b/a so that you can make a reasonable graph of R vs. b/a . Demonstrate that in the limit $b/a \gg 1$ the resistance R approaches $R = \rho b/a$ where ρ is the sheet resistivity of the conductor (i.e. the ratio of the electric field \mathbf{E} to the sheet current density \mathbf{K} .)

2. Now, returning to a square sample, generalize to the case where the resistivity of the metal is not isotropic. In other words, the (2D) current density \mathbf{K} is related to the electric field \mathbf{E} in this way:

$$E_x = \rho_{xx} K_x$$

$$E_y = \rho_{yy} K_y$$

and $\rho_{xx} \neq \rho_{yy}$. (Recall that these are 2D resistivities, the plate is assumed very thin and the current uniform through its thickness.)

Set up the boundary value problem as in class, using $\mathbf{K} = \mathbf{z} \times \nabla\phi$. Let the x -axis correspond to the direction from A to B in the drawing.

Determine the resistance, $R = V_{cd} / I$, of the square in terms of the two resistivities.

Work out R numerically (to reasonable accuracy) in two specific cases: $\rho_{xx} = 1$, $\rho_{yy} = 0.2$ and $\rho_{xx} = 0.2$, $\rho_{yy} = 1$. How does the ratio of the R 's in these cases compare to the ratio

of the ρ 's? Explain in physical terms the origin of the difference. Sketch the current flow patterns in each case.

3. In the presence of a magnetic field, the relation between current density and the electric and magnetic fields present becomes

$$\mathbf{J} = \sigma (\mathbf{E} + \mathbf{v} \times \mathbf{B})$$

with σ the conductivity (here isotropic) and \mathbf{v} the drift velocity of the carriers, and $\mathbf{J} = ne\mathbf{v}$. Considering only current flow in a 2D plane (with \mathbf{B} perpendicular to that plane), show that \mathbf{J} and \mathbf{E} are related through a 2x2 conductivity tensor:

$$\begin{pmatrix} J_x \\ J_y \end{pmatrix} = \begin{pmatrix} \sigma_{xx} & \sigma_{xy} \\ \sigma_{yx} & \sigma_{yy} \end{pmatrix} \begin{pmatrix} E_x \\ E_y \end{pmatrix}$$

Evaluate the components of the conductivity tensor in terms of σ , B , n , e , and v .

Evaluate the associated resistivity tensor and demonstrate that, at non-zero magnetic field, one can have both σ_{xx} and the resistivity ρ_{xx} simultaneously vanish. Discuss the physical reasons for why this is not crazy. (For this last, you may simply assume that there are systems in which no electron scattering occurs.)

4. Using all of the above, describe the current flow pattern in a square sample at zero and high magnetic field.

5. A point dipole \mathbf{m} is embedded in a linear magnetic material with susceptibility χ_m which extends throughout space. Find the magnetic field everywhere.

6. Griffiths 6.13