Critical mass

- How much $^{239}$Pu is needed for a sustained chain reaction?

**First: Historical perspective:**

Heisenberg $\leq 1941$ or so:
- As big as a grapefruit.

Heisenberg $\leq 1942$ or so:
- 8 tons.

**The sad story**

In Los Alamos, experiments were carried out with a 6.7 ton $^{239}$Pu core. In one experiment a neutron reflecting brick fell on the core, making it go critical. Harry Byghtian died of radiation poisoning within
a month. The accident was on 21st of August 1945. Incidentally, a few days after the victory parade on Times Square.

Louis Slotin was killed by the same core going supercritical on May 1st 1946. He was carrying out the "tickling the dragon's tail" experiment.
A mock-up of the “Tickling the Dragon’s Tail” experiment that killed Louis Slotin

Taken from:

CURSE OF THE DEMON CORE!

Stanley Wojculewski | November 3, 2013 | Awesome History | No Comments

Critical mass: general info

\[ n \rightarrow n \rightarrow n \rightarrow n \]

Average yield of fission: \( N = 2.5 \)

Energy of neutrons emitted:

\[ E_n \sim 1 \text{ MeV} \]
So 1% of rest mass — the neutrons are relativistic!

Cross sections:
measured in barns = \( 10^{-28} \text{ m}^2 \)

Probably conceived by: "This Uranium is as big as a f...ing barn!"

For Fast Neutrons: \( \Sigma_{sc} \approx 56 \)

For Scattering off \( ^{239}\text{Pu} \)

Fission cross section:
\( \Sigma_f \approx 26 \)

Note: for thermal neutrons, \( 0_{\text{f}} \text{ too!} \)
Calculation

Assume at first no scattering.

Either a neutron leaves the sphere, or it fissions a $^{239}$Pu.
Probability for fission:

\[ p = \left( \frac{\sigma}{m} \right) \cdot \sigma_f \cdot p_0 \]

Volume density of fissile nuclei

(distance travelled in core) \sim R

We need that the yield of neutrons will make up for lost neutrons:

\[ p \cdot N > 1 \]

So:

\[ \left( \frac{\sigma}{m} \right) \cdot R \cdot \sigma_f \cdot N = 1 \]
\[
R_{\text{crit}} = \frac{1}{(8/N \cdot G_N) \cdot \text{cm}}
\]

Calculate:

\[
S = 10^5 \text{ g/m}^3
\]

\[
m = 239 \cdot 1.6 \cdot 10^{-27} \text{ kg}
\]

\[
a \approx 4.10^{-25} \text{ kg}
\]

Then:

\[
R_{\text{crit}} = \frac{4}{10^{429} \text{ m}^3 \cdot 2.10^{-28} \text{ m}^2 \cdot 2.5} = \text{8 cm}
\]

\[
\frac{1}{\text{mass}}
\]

\[
m_{\text{crit}} \approx \frac{4 \pi}{3} \cdot 10^4 \text{ g/m}^3 \cdot (0.08 \text{ m})^3
\]

\[
a \approx 20 \text{ ug}
\]

According to Wikipedia: 11 \text{ ug}.
Question: how much would diffusion help?

Well, the mean free path is:

\[ l_0 \sigma c \left( \frac{\rho}{m} \right) = 1 \]

\[ l = \frac{1}{(\pi/2) \cdot 6 \sigma c} = \frac{4}{10^{29} \ V_m^3 \cdot 5 \cdot 10^{-28} \ m^2} \]

\[ = 8 \text{ cm} \ldots \]

Same as the radius...

Last question: Bomb Yield?

When does a chain reaction stop? When there is no critical mass any longer.
Each nucleus split:

\[ \Delta E = 200 \text{ MeV}. \]

So:

\[ \Delta E \cdot \frac{(m_0 - m_{\text{crit}})}{m_p} = E_{\text{total}} \]

Try

\[ \frac{m_0}{m_{\text{crit}}} \approx 1.2 \]

\[ E_{\text{total}} \approx 0.2 \frac{m_{\text{crit}}}{m_p}, \Delta E = 0.2 \cdot \frac{200}{4.1 \times 10^{-25}} \cdot 200 \text{ MeV} \]
\[
E_{\text{total}} = 2 \cdot 10^{27} \text{ MeV}
\]
\[
= 2 \cdot 10^{27} \times 1.6 \cdot 10^{-13} \, \text{J}
\]
\[
= 3 \cdot 10^{14} \, \text{J} = 300 \, \text{TJ}
\]

<table>
<thead>
<tr>
<th>Bomb</th>
<th>Yield (kt TNT)</th>
<th>Yield (TJ)</th>
<th>Notes</th>
</tr>
</thead>
<tbody>
<tr>
<td>Davy Crockett</td>
<td>0.01</td>
<td>0.042</td>
<td>Variable yield: tactical nuclear weapon—mass only 23 kg (51 lb), lightest ever deployed by the United States (same warhead as Special Atomic Demolition Munition and GAR-11 Nuclear Falcon missile).</td>
</tr>
<tr>
<td>Hiroshima's &quot;Little Boy&quot;</td>
<td>13–18</td>
<td>54–75</td>
<td>Gun type uranium-235 fission bomb (the first of the two nuclear weapons that have been used in warfare).</td>
</tr>
<tr>
<td>Nagasaki's &quot;Fat Man&quot;</td>
<td>20–22</td>
<td>84–92</td>
<td>Implosion type: plutonium-239 fission bomb (the second of the two nuclear weapons used in warfare).</td>
</tr>
<tr>
<td>W76 warhead</td>
<td>100</td>
<td>420</td>
<td>Twelve of these may be in a MIRVed Trident II missile; treaty limited to eight.</td>
</tr>
<tr>
<td>W87 warhead</td>
<td>300</td>
<td>1,300</td>
<td>Ten of these were in a MIRVed LGM-118A Peacekeeper.</td>
</tr>
<tr>
<td>W88 warhead</td>
<td>475</td>
<td>1,990</td>
<td>Twelve of these may be in a Trident II missile (treaty limited to eight).</td>
</tr>
<tr>
<td>Ivy King device</td>
<td>500</td>
<td>2,100</td>
<td>Most powerful pure fission bomb, 60 kg uranium, implosion type.</td>
</tr>
<tr>
<td>B83 nuclear bomb</td>
<td>variable</td>
<td>Up to 1.2 megatonnes of TNT (5.0 PJ); most powerful US weapon in active service.</td>
<td></td>
</tr>
<tr>
<td>B53 nuclear bomb</td>
<td>9,000</td>
<td>38,000</td>
<td>Was the most powerful US bomb until 2010; it was not in active service for many years before 2010, but during that time, 50 were retained as part of the &quot;Hedge&quot; portion of the Enduring Stockpile until completely dismantled in 2011. A variant of the two-stage B61 is the B53 replacement in the bunker-busting role; the B53 was similar to the W-53 warhead that has been used in the Titan II Missile; decommissioned in 1987.</td>
</tr>
<tr>
<td>Castle Bravo device</td>
<td>15,000</td>
<td>63,000</td>
<td>Most powerful US test.</td>
</tr>
<tr>
<td>EC17/Mk-17, the EC24/Mk-24, and the B41 (Mk-41)</td>
<td>various</td>
<td>Most powerful US weapons ever: 25 megatonnes of TNT (100 PJ); the Mk-17 was also the largest by size and mass: about 20 short tons (18,000 kg); The Mk-41 or B41 had a mass of 4800 kg and yield of 25 Mt, this equates to being the highest yield-to-weight weapon ever produced; all were gravity bombs carried by the B-36 bomber (retired by 1957).</td>
<td></td>
</tr>
<tr>
<td>Operation Castle nuclear test series</td>
<td>48,200</td>
<td>202,000</td>
<td>The highest-yielding test series conducted by the US.</td>
</tr>
<tr>
<td>Tsar Bomba device</td>
<td>50,000</td>
<td>210,000</td>
<td>USSR, most powerful nuclear weapon ever detonated, yield of 50 megatons, (50 million tons of TNT). In its &quot;full&quot; form (i.e. with a depleted uranium tamper instead of one made of lead) it would have been 100 megatonnes of TNT (420 PJ).</td>
</tr>
</tbody>
</table>

Wikipedia: Nuclear bomb yield.