Problem: How far does an airstream out of a flat go?

Initial velocity: \( V_0 = 5 \text{m/s} \)

Aperture radius: \( r \approx 10^{-3} \text{m} \)

Attempted Solution:

Main culprit: viscosity.

\[
\frac{\partial p}{\partial t} = \eta A \frac{\partial V}{\partial x} \]

\( A \)- contact area between layers

\( \frac{\partial p}{\partial t} \)- viscous force

\( V_x \)- velocity profile

\( \eta \)- viscosity coefficient.

Flat: assume cylindrical symmetry.

Looking out a small ring:

\( \frac{\partial}{\partial r} \)

Let's write viscosity equations:

\[
dF = (1/2 \cdot 2\pi r) \frac{\partial V}{\partial r} \cdot \eta \]

at contact with outside \((or\ inside)\)

So taking both sides into account:

\[
dF_{\text{total}} = dF_{\text{fric}} + \frac{1}{2} \pi r \eta \frac{\partial V}{\partial r} \cdot dr \]
Now we assume that pressure everywhere is $P = 10^5 N/m^2$. So this is the only force on the rig:

$$F = ma$$

But $F = ma$?

$$m \frac{dv}{dt} = 2\pi rdrdz \sigma_m \frac{dv}{dt}$$

But $\frac{dv}{dt}$ must be a material derivative.

Velocity can change as:

$$\frac{\partial v}{\partial t}$$

But also, as $z$ changes:

$$\frac{\partial v}{\partial z} \cdot dz = v \cdot \frac{\partial v}{\partial z}$$

And:

$$\frac{dv}{dt} = \frac{\sigma_m}{\rho} + v \frac{\sigma_m}{\rho}$$

Therefore second law reads:

$$(2\pi rdrdz \sigma_m) \left( \frac{\partial v}{\partial t} + v \frac{\partial v}{\partial z} \right) = \mu dm dr \frac{\partial}{\partial z} \left( \frac{\sigma_m}{\rho} \right)$$

Now more assumptions:

$\sigma_m$ constant (otherwise we are really sad)

$\frac{\partial v}{\partial t} = 0$ since we assume a steady state.

$$J_m v \frac{\partial v}{\partial z} = \gamma \frac{1}{r} - \frac{e}{r} \left( -\frac{\partial v}{\partial r} \right)$$

Looks nasty. Maybe set of valid

$$v = R(r) \cdot S(z)$$
Let's see:

\[ \frac{g_m}{z} = \frac{R_{\text{W}}}{z}, \quad \frac{g}{z} = \frac{R_{\text{W}}}{z} \]

and:

\[ \frac{g}{z} = \frac{R_{\text{W}}}{z}, \quad \frac{g}{z} = \frac{R_{\text{W}}}{z} \]

Looks hopeless...

Let's continue:

\[ \frac{S}{z} = \frac{1}{g_{\text{W}}}, \quad (z-2) \]

But here:

\[ \frac{z}{z-2} = \frac{1}{R_{\text{W}}}, \quad (z-2) \]

What is \( R_{\text{W}} \)?

Still... what is \( R_{\text{W}} \)?
Get it from initial conditions:

\[ V_o = \frac{2 \cdot 0.4 \cdot \frac{y}{r_o^2}}{\frac{5}{3}} \]

\[ 2_o = \frac{V_o \cdot r_o^2}{4} \cdot \frac{3}{5} \]

\[ \rho_m = \frac{p}{k T} \cdot m = \frac{10^5}{1.8 \cdot 10^{-23} \frac{J}{K} \cdot 200 K} \]

\[ \approx \frac{1}{4} \cdot 10^{26} \cdot 50 \cdot 10^{-12} \cdot \left[ \frac{kg}{m^3} \right] \]

\[ \approx 1 \text{ kg/m}^3 \]

\[ \gamma = 3 \cdot 10^{-5} \text{ kg} \cdot \text{m} \cdot \text{s}^{-2} \]

Under our conditions: \( r_o = \frac{1}{10} \text{ m} \):

\[ \varepsilon_o = 5 \alpha_m \cdot 10^{-8} \text{ m}^2 \cdot \frac{1}{3} \cdot 10^5 \frac{\text{ kg}}{\text{ m}^2} \cdot \frac{1}{4} \]

\[ \approx 0.01 m \]

For \( r_o = 10^{-3} \text{ m} \):

\[ \delta_m \]

Simpler argument:

\[ \frac{y}{\gamma} = \cdots \left( \frac{m}{\zeta} \right) \]

Diffusion:

How about, how long for air from outside the stream to diffuse in:

\[ \sqrt{D \cdot t} \]

Try:

\[ t \cdot V_o = \varepsilon_o = \frac{V_o \cdot r_o^2}{D} \text{ lim} \]
In fact:

$$D \sim V_t \cdot l_{\text{mft}}$$

So:

$$z_0 \sim \frac{V_o}{V_t} \cdot \frac{z_0}{l_{\text{mfp}}} \sim \frac{V_o}{V_t} = \frac{R_o}{l_{\text{mfp}}}$$

That's simple.