Ph 50 - Balloon-Assisted Launches

- A new kind of tinkering project: using weather balloons to photograph the Earth from near space

→ lots of cool pictures!

Here's an idea: could we maybe take this one step further and use a balloon for a satellite launch?

"Practical" application: the N-prize

First, how high can a balloon go?

\[ T_1 \approx 0 \]

\[ P', V', T', N \]

\[ \text{will eventually rupture!} \]

\[ \text{atm} \rightarrow T_0 \]

\[ P_0, V_0, T_0, N \]
1) Look at the "best" balloon
   → Felix Baumgartner's balloon

Some facts:
- Filled with He
- 180,000 ft³ at launch
- 30 x 10⁶ ft³ at max height
- Balloon mass 1680 kg
- Capsule mass 1315 kg

- Estimate height based on buoyancy

Archimedes: \[ F_B = m_{\text{displaced}} \cdot g = \rho V_{\text{disp}} \cdot g \] (Assume \( g \approx \text{const} \approx 10 \text{ m/s}^2 \))

Sp² at max height, balloon is at equilibrium.

I.e. \((m_{\text{balloon}} + m_{\text{capsule}}) \cdot g = \rho V_{\text{disp}} \cdot g\) (neglect He mass)

We know \( V_{\text{disp}} = 3 \times 10^7 \text{ ft}^3 \approx 8.5 \times 10^5 \text{ m}^3 \)

Q: What is \( \rho \)?
Wiki: \( \log p = -\frac{3}{50} h - 3 \log \left( \frac{g}{cm^2} \right) \) \( h \) in km

\[ p = 10^{-\frac{3}{50} h - 3} \frac{g}{cm^3} \]

\[ p = 10^{-\frac{3}{50} h} \frac{kg}{m^3} \]

So \( M_{\text{Tot}} = 10^{-\frac{3}{50} h} \frac{kg}{m^3} \cdot V \)

or \( h = -\frac{50}{3} \log \left( \frac{M_{\text{Tot}}}{V} \right) \) \( \text{km} \)

\[ \approx -\frac{50}{3} \cdot \log \left( \frac{1680 + 1315}{850000} \right) \]

\[ \approx 40 \text{ km} \]

Red Bull website: \( 36.969 \text{ km} \) on \( \checkmark \)
Now consider launching a satellite.

**NASA proposal:** launch from 35 km

payload 10 kg

orbit at 200 km

How high is this?

700 km

ISS: 330 - 410 km

Thermo

80 km

Meso

50 km

Strato

12 km

Troposphere

How much fuel can you save?

A very crude model:

Circular orbit: \( E = \frac{1}{2} U \)

\[ E = \frac{-GMm}{2(R_1 + h_2)} \]

payload \( m = 10 \text{ kg} \)

Regular launch: \( \Delta E = \frac{-GMm}{2(R_1 + h_2)} + \frac{GMm}{R_1} \)

\[ \Delta E = \frac{GMmR^2}{a^2} \left( 1 - \frac{R_1}{2(R_1 + h_2)} \right) \]
\[ \Delta E = \frac{10 \, \text{m}}{s^2} \cdot 10 \, \text{kg} \cdot 6.4 \times 10^6 \, \text{m} \left( 1 - \frac{6.4 \times 10^6 \, \text{m}}{2 \left( 6.4 \times 10^6 \, \text{m} + 2 \times 10^5 \, \text{m} \right)} \right) \]

\[ \approx 3.3 \times 10^8 \, \text{J} \]

Balloon: \[ \Delta E = g m R \left( \frac{R}{R + h_1} - \frac{R}{2(R + h_2)} \right) \]

\[ = 3.2 \times 10^8 \, \text{J} \]

\[ \frac{0.1 \times 10^8 \, \text{J}}{3.3 \times 10^8 \, \text{J}} \approx 3 \% \quad \text{pretty dismal} \]

But, this model is very crude... it neglects the mass of the propellant which is needed to elevate the payload.

\[ \rightarrow \text{mass needed should scale quadratically at least as a function of height, i.e. go higher \rightarrow more propellant \rightarrow need to lift this extra propellant} \]

Setup:

\[ r_i, \quad m_f, \quad m_p(r_i) = 0 \]

Q: What is \( m_p(r_i) \) for a given \( r_i, r_0, m_f \)?

payload mass \( m_f = \)

\[ r_0 \quad \text{propellant mass} \quad m_p(r_0) \]
Thrust: \[ T = - \dot{m}_p v_{rel} \rightarrow \text{assume const. } \dot{m}_p, \ v_{rel} \text{ so } T \text{ const} \]

Newton: \[ T + F_g = (m_p + m_\#) \frac{dv}{dt} \\
F_g = - \frac{g \beta^2 (m_p + m_\#)}{r^2} \]

Now, \[ \dot{m}_p = \frac{d\dot{m}_p}{dt} = \frac{d\dot{m}_p}{dr} \frac{dr}{dt} = - \frac{I}{v_{rel}} \text{ const } \rightarrow \frac{dr}{dt} = - \frac{I}{v_{rel}} \frac{1}{d\dot{m}_p/dr} \]

and \[ a = \frac{dv}{dt} = \frac{dv}{dr} \frac{dr}{dt} = \frac{dv}{dr} \left( - \frac{I}{v_{rel}} \frac{1}{d\dot{m}_p/dr} \right) \]

\[ \Rightarrow T - \frac{g \beta^2}{r^2} (m_p + m_\#) = (m_p + m_\#) \frac{dv}{dr} \left( - \frac{I}{v_{rel}} \frac{1}{d\dot{m}_p/dr} \right) \]

Let \[ m_{tot} = m_p + m_\#, \quad \frac{dm_{tot}}{dr} = \dot{m}_p \]

\[ \Rightarrow \left( \frac{1}{m_{tot}} - \frac{g \beta^2}{r^2} \right) \frac{dm_{tot}}{dr} = - \frac{1}{v_{rel}} \frac{dv}{dr} \]

\[ \int_{r_0}^{r_1} \frac{1}{m_{tot}} \frac{dm_{tot}}{dr} dr - \int_{r_0}^{r_1} \frac{g \beta^2}{r^2} \frac{dm_{tot}}{dr} dr = - \frac{1}{v_{rel}} \int_{r_0}^{r_1} \frac{dv}{dr} dr \]

\[ \text{approximate } \frac{\beta^2}{r^2} \approx 1 \quad \Rightarrow \quad \int_{r_0}^{r_1} \frac{1}{m_{tot}(r_1)} \frac{dm_{tot}}{dr} dr - \frac{g}{r} (m_{tot}(r_1) - m_{tot}(r_0)) = - \frac{1}{v_{rel}} (v(r_1) - v(r_0)) \]

Let's consider just lifting the payload up to 200km to be able to go farther, i.e., set \( v(r) = v(r_0) = 0 \)

Also, notice \[ m_{tot}(r_1) - m_{tot}(r_0) = m_p(r_1) + m_\# - m_p(r_0) - m_\# \]
\( m_p(r_i) = 0 \)

\[
\ln \left( \frac{m_f}{m_p(r_o) + m_f} \right) + g \frac{m_p(r_o)}{r} = 0
\]

\[
\Rightarrow \frac{m_f}{m_p(r_o) + m_f} = e^{- \frac{g}{r} m_p(r_o)}
\]

- During a launch, astronaut experience about 1 to 3 g's
- So, let's say \( g \frac{m_p(r_o)}{r} \approx 2 \)

\[
\Rightarrow \frac{10}{m_p(r_o) + 10} \approx e^{-2}
\]

\[
10 e^2 - 10 \approx m_p(r_o) \approx 65 \text{ kg}
\]

But wait... how did we get a mass out ?? We never said how high we're launching from...

\[
\Rightarrow \text{This is because we made the approximation } \frac{g}{r} \approx 1
\]

\[
\Rightarrow \text{really, } 1 \leq \frac{g}{r} \leq 0.97 \text{ so at best we may get about 3% more mass over a launch from ground to 400 km}
\]

\[
\Rightarrow \text{clearly, we don't save much propellant with a balloon.}
\]

\textbf{What gives?}
Let's read the report more carefully...

"A high altitude launch will reduce the air drag acting on the vehicle, reducing propellant needs."

Alas... air resistance!

**Homework:**

1. Come up with a model for the drag force due to air on a rocket.

2. (Bonus) See how much more propellant you need to launch a rocket from Earth to Earth + 200km with payload 10 kg, not neglecting drag.

(I haven't done this one yet, but should be pretty similar to the calculation we did, just now \( \Sigma \text{Force} = T + F_g + F_d \))